

The supplementary materials

The floating process of oil droplets and bubbles in water is simultaneous. Therefore, the theory of the SCF work can be described by the same set of equations. The oil droplet is taken as an example for calculation below.

The buoyancy of oil droplet in water is calculated as:

$$F_s = \frac{1}{6} \pi d_p^3 (\rho_w - \rho_p) \cdot g \quad (1)$$

where F_s is the buoyancy of oil droplet, d_p is the particle size of the oil droplet, ρ_w is the density of the water, ρ_p is the density of oil droplet, and g is the acceleration of gravity, respectively.

The resistance of oil droplet in water is calculated as:

$$F_f = -\lambda \left(\frac{\pi}{4} d_p^2 \right) \left(\frac{u^2 \rho_w}{2} \right) \quad (2)$$

where F_f is the resistance of oil droplet, λ is the resistance coefficient, and u is the floating speed of oil droplet in water, respectively.

When the rising speed of the oil droplet in the water does not change, the acceleration of the oil droplet is zero. At this time, the buoyancy of the oil droplet is the same as the resistance, then:

$$\frac{1}{6} \pi d_p^3 (\rho_w - \rho_p) \cdot g = -\lambda \left(\frac{\pi}{4} d_p^2 \right) \left(\frac{u^2 \rho_w}{2} \right) \quad (3)$$

$$Re = \frac{\rho_w u d_p}{\mu} \quad (4)$$

$$u = \frac{g d_p^2 (\rho_w - \rho_p)}{18 \mu} \quad (5)$$

where Re is Reynolds number of oil droplet, μ is the viscosity of water.

According to the relevant knowledge of hydrodynamics, under different laminar and turbulent conditions, the relationship between drag coefficient C_D and Reynolds number Re is a piecewise function.

At the laminar flow zone ($Re < 1$) :

$$C_D = \frac{24}{Re} \quad (6)$$

At this time, the rising process of bubbles in water follows Stokes law

$$u_g = \frac{gd_g^2(\rho_w - \rho_g)}{18\mu} \quad (7)$$

At Turbulent transition zone ($1 < Re < 300$) :

$$C_D = \frac{18.5}{Re^{0.6}} \quad (8)$$

At this time, the rising speed of bubbles in water is

$$u_g^{1.4} = \frac{1}{13.9} \frac{gd_g^{1.6}(\rho_w - \rho_g)}{\mu^{0.6}\rho_w^{0.4}} \quad (9)$$

In the turbulent region ($300 < Re < 4000$), the resistance of the bubble increases with the increase of Reynolds number Re . At this time, the rising velocity of the bubble can be calculated by Mendelson empirical formula:

$$u_g = \left[\frac{2\sigma}{d_g\rho_w} + \frac{gd_g}{2} \right]^{1/2} \quad (10)$$

The above analysis is based on the equation that the bubble is a standard sphere and does not change during the upward floating process. When the particle size of the bubble is large and the shape will change during the upward floating process, the upward floating speed of the bubble will also change. The dimensionless numbers that affect the morphological changes in the process of bubble floating include bond number and Morton number:

bond number:

$$B_o = \frac{g(\rho_w - \rho_g)d_g^2}{\gamma^3} \quad (11)$$

Morton number:

$$M_o = \frac{g(\rho_w - \rho_g)\mu^4\rho_w^2}{\gamma^3} \quad (12)$$

where ρ_w is the density of the aqueous phase, μ is the viscosity of the aqueous phase, g is the acceleration of gravity, and γ is surface tension.

In the actual flotation process, the temperature of oilfield produced water entering the separator was basically unchanged and set to at 50 °C. The density of water phase,

gas phase and oil phase were 988 kg/m^3 , 1.043 kg/m^3 and 955 kg/m^3 , respectively. The water phase viscosity was $0.5494 \text{ mPa}\cdot\text{s}$.

Based on above formula, the rising velocity of oil particles can be calculated. The rising velocity of oil droplets gradually increases with the increase of oil droplets size. Reynolds number is a basic parameter to estimate the flow pattern [1]. When the oil droplets at particle sizes of 100, 200 and $300 \text{ }\mu\text{m}$, the rising velocity is 0.33 mm/s , 1.32 mm/s and 2.65 mm/s , and the Reynolds number is 0.06, 0.47 and 1.43, respectively. All Reynolds number of oil droplet is below 4.5, which indicate that oil droplets in the laminar flow area and transition area. The free floating of bubbles will have little disturbance to the fluid.

In the apparatus, the aeration disc has $50 \text{ }\mu\text{m}$ uniform diameter micropores. Under the action of aeration pump, bubbles generate continuously and evenly, and enter into the oil produced water. The bubble size is affected by the rising velocity, pressure and surface tension in the rising process. Some bubbles will gather to form large bubbles, and some bubbles will burst. The bubble particle size is expected to be 10 to $200 \text{ }\mu\text{m}$. In bubble column reactor, the main influencing parameters of bubbles rising velocity are liquid superficial velocity and gas superficial velocity [2]. Increasing gas flow rate leads to an increasing strength of the recirculation, which is expressed by higher shear rates in the vertical liquid velocity [3].

When the liquid volume flow at the liquid inlet is $1.35 \text{ m}^3/\text{h}$, the Reynolds number of oil produced water is 9346, which indicate the inflow is at turbulence. When the oil produced water enters into the apparatus, the flow rate decreases. The rising speed of bubbles in water could be obtained by Equation.4. The laminar flow region and transition region were marked in the Fig.S1 according to the change of Reynolds number.

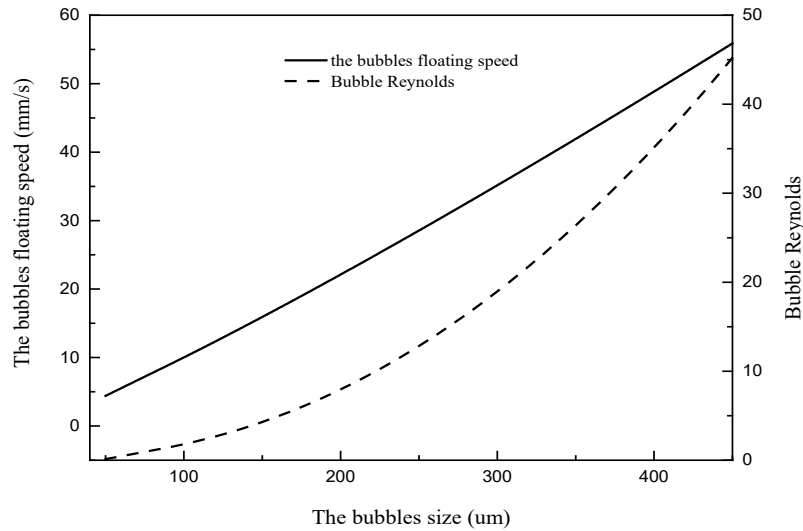


Fig.S1 The relationship between bubble floating velocity in water and bubble particle size

As shown in Fig.S1, the rising speed of bubbles gradually increased with the increased bubble particle size. The rising speed of bubbles changed from laminar flow zone to turbulence transition zone with the increase of particle size, with the particle size at 100 μm , and the maximum rising speed at about 10mm/s. When the particle size of bubbles was greater than 84 μm , the stable rising speed of the bubble would cause the Reynolds number of the bubble between 1 and 50, which was in the transition zone. The maximum Reynolds number of the bubble was still small and the disturbance of the fluid caused by the bubble floating was also very small.

Since the floating process of oil droplets is similar to that of bubbles, the variation curve of the floating speed of oil droplets with the particle size of oil droplets can also be calculated and shown in supplements. Similar to the bubble floating process, the floating speed of oil droplets also gradually increases with the increase of particle size. The floating velocity of oil droplets gradually increases with the increase of oil droplets size. When the oil droplets at particle sizes of 100, 200 and 300 μm , the rising velocity is 0.33mm/s, 1.32 mm/s and 2.65 mm/s, and the Reynolds number is 0.06, 0.47 and 1.43, respectively. All Reynolds number of oil droplet is below 4.5 and oil droplets is at the laminar flow area and transition area. The free floating of bubbles will have little disturbance to the fluid. In terms of oil concentration in the influent and effluent at a certain time, the removal rate increased with the increased of particle size, indicating that the apparatus is suitable for removing large particle size oil droplets. Similar to the bubble rise analysis, the relationship bubble floating velocity in water with bubble

particle size is obtained and can be seen in Fig.S2. The laminar flow region and transition region are marked in the figure according to the change of Reynolds number.

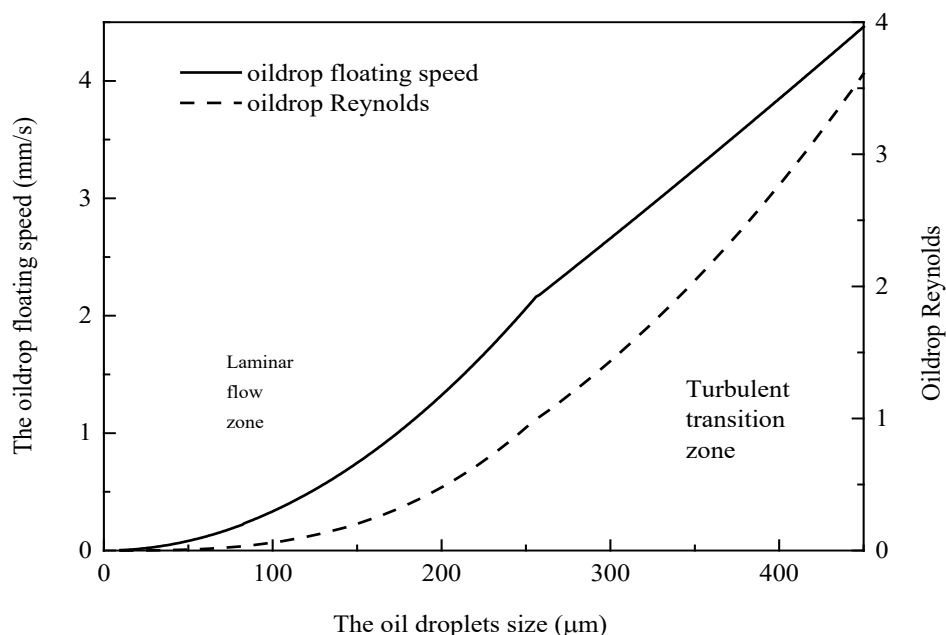


Fig.S2 The relationship between oil drop floating speed in water and oil droplets size

Oil phase which determined by oil droplet size within the water phase could be classed as free oil ($>150\ \mu\text{m}$), dispersed ($20\text{ to }150\ \mu\text{m}$), emulsified ($5\text{ to }20\ \mu\text{m}$), and soluble ($<5\ \mu\text{m}$) [4]. Free oil form mainly exists in the influent oil field-produced water and emulsified oil form mainly exist in the effluent oil field-produced water in this study. The particle size distribution of the influent and effluent oil produced water was shown in Table.S1.

Table.S1 The oil droplets distribution in the oil produced water

	Total	$<20\mu\text{m}$	$20\text{-}150\mu\text{m}$	$>150\mu\text{m}$
Influent	100%	11.79%	15.47%	72.74%
Exfluent	100%	59.94%	39.90%	0.16%
Removal rate	92.43%	61.52%	80.48%	99.98%

When the influent oil concentration was 1193.26 mg/L , the particle size of the oil droplets in D50 was $250.6\ \mu\text{m}$, and the maximum particle size of the oil droplets was $900.9\ \mu\text{m}$. When the effluent oil concentration was 90.32 mg/L , the particle size of D50 oil drops was $13.7\ \mu\text{m}$ and the maximum particle size was $219.3\ \mu\text{m}$, indicating that

part of the emulsified oil with larger particle size had been removed by primary filtration and only small particle size emulsified oil existed in oil produced water. The removal rate curve of oil particles at a certain time is shown in Fig. S3:

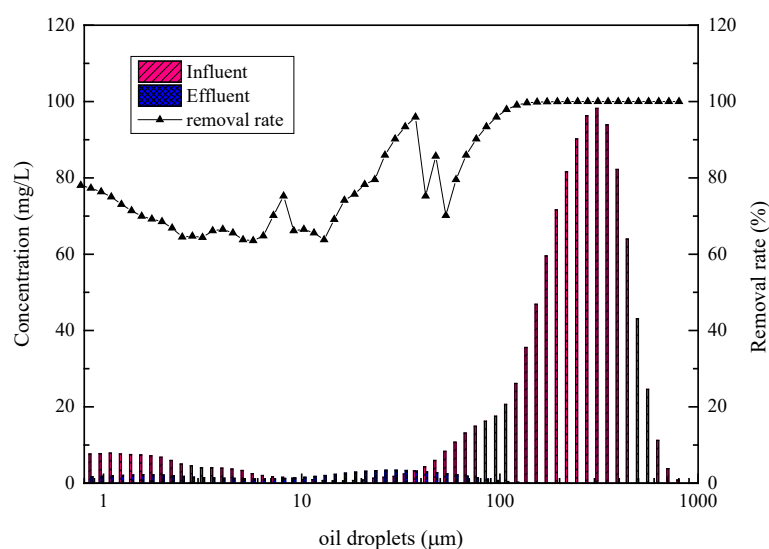


Fig.S3 Oil droplets size distribution and removal rate

- [1] Xu, X., et al., Bubble-separation dynamics in a planar cyclone: Experiments and CFD simulations. *AIChE Journal*, 2018. 64(7): p. 2689-2701.
- [2] Leonard C, Ferrasse J, Lefevre S, Viand A, Boutin O, Bubble rising velocity and bubble size distribution in columns at high pressure and temperature: From lab scale experiments to design parameters. *Chemical Engineering Research and Design*, 2021. 173: p. 108-118.
- [3] Panjipour, R., M. Karamoozian and B. Albijanic, Bubble size distributions in gas–liquid–solid systems and their influence on flotation separation in a bubble column. *Chemical Engineering Research and Design*, 2021. 167: p. 96-106.
- [4]. Edzwald J. K., Principles and applications of dissolved air flotation. *Water Science and Technology*, 31(3-4), 1995:1-23.